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Dynamic tariffs, demand response, and regulation in retail electricity markets☆

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ABSTRACT

Greater penetration of renewables in electricity generation will result in high variability in residual demand (demand net of renewable generation); this will further challenge the stability and flexibility of power systems. One possible solution is demand response, which is usually achieved through dynamic tariffs that offer consumers financial incentives to shift or reduce peak load to off-peak periods. We construct a two-stage dynamic game to model the retail market, in which the retailer sets dynamic tariffs to maximize profit, and consumers respond to the prices. Using the Irish smart metering data as model inputs, we find that in our baseline scenario, the dynamic tariff would generate for the retailer an additional €7.35 of annual profit from a representative Irish household. With market regulations, the dynamic tariff will benefit consumers and retailers alike. We also find that the interaction between demand-side management stimuli and market regulation can further reduce consumer-level electricity demand, increase retail profit, and lower consumers' electricity bills.

1. Introduction

Meeting European Union's (EU's) and United Kingdom's (UK's) net-zero carbon targets will result in high levels of renewable energy (wind and solar) production, displacing fossil generation. Given that the generation of renewable energy is variable and difficult to predict, this brings challenges for power system stability and flexibility. One possible solution is demand response (DR), which seeks to balance supply and demand by providing electricity consumers with financial incentives to shift reduce or shift peak load to off-peak periods. Bradley et al. (2013) argued that despite the controversy among researchers about DR's costs and benefits, DR can produce net positive economic welfare in the electricity market.

One of the most efficient solutions to DR is offering consumers dynamic electricity tariffs such as time-of-use (TOU),[1] critical peak pricing (CPP),[2] and real-time pricing (RTP).[3] Dynamic tariffs were initially implemented in industrial sectors to address large and controllable loads, while incurring relatively low costs per control point (Roos, 1998). Starting in 2003, a number of pilot studies examined the impact of household-level dynamic tariffs. Faruqui and Sergici (2010), for example, surveyed a number of pilot studies, two of which considered the RTP tariff as a treatment group. Both of these RTP pilot studies demonstrated that RTP tariffs outperform flat tariffs in terms of peak demand reduction and lowering household-level electricity bills.

However, as most households are risk averse and are unwilling to be exposed to volatile electricity prices (Shirani et al., 2020; Chamaret et al., 2020), currently retailers usually offer flat tariffs that charge a fixed rate for every kilowatt-hour (kWh) of electricity purchased. Shirani et al. (2020), therefore, argued that dynamic tariffs should target voluntary consumers.

Dynamic tariffs can be attractive from the perspective of the retailer. This is because dynamic tariffs allow retailer to align retail prices with spot market prices (SMPs), thereby transferring some retailer-centred risk to customers. As noted by Nilsson et al. (2018), dynamic tariffs constitute the most effective strategy by which to increase demand flexibility. Nojavan et al. (2017), in studying the impact of different price schemes on retail profit, suggested that RTP tariffs can result in higher retail profit.

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[1] Under a TOU tariff, a day is separated into several periods, and each period has different prices. TOU tariffs are generally fixed or dynamic TOU, according to whether prices differ for the same period across days.

[2] Under a CPP tariff, only peak prices vary across days; peak prices are usually higher than prices in other periods.

[3] Under a RTP tariff, prices are varying hourly or half-hourly.

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The increase in the market penetration of variable renewable energy, with the attendant increase in imbalance volumes and related costs, has created considerable potential for DR aligned with dynamic tariffs (Balijepalli et al., 2011; Pina et al., 2012). In addition, given that the marginal fuels used during peak periods are those with higher carbon intensity, peak load shifting can help reduce greenhouse gas emissions. Holland and Mansur (2008) found that the implementation of the RTP tariff reduces greenhouse gas emissions in US regions where peak demand is met mainly by oil-fired capacity.

Dynamic tariffs cannot be effectively implemented without the help of smart metres which record energy consumption in each (half-)hourly period and communicate with energy suppliers and network companies. As a way of reducing retail costs and encourage consumers to pay more attention to the energy they use, governments are committed to delivering programmes to encourage smart metering implementation. These programmes are expected to have significant economic benefits in the long term as renewable energy and electric vehicles become more widespread. With the roll-out of smart metres, retailers are able to learn how consumption responds to varying prices, and consumers receive information about energy prices ahead of time via texts, emails or in-house displays, ensuring the transparency of dynamic tariffs.

In this article, we leverage a two-stage dynamic game to derive time-varying retail electricity prices that maximize retail profit conditional on households' decisions regarding how much electricity to consume. Using the flat tariff as a benchmark, we investigate the effect of dynamic tariffs on the retailer profit, the household electricity bill and electricity demand, as well as the market gains — the sum of the increase in the retailer profit and the reduction in the household electricity bill. To provide households with strong incentives to choose dynamic tariffs, the model guarantees them paying no more bills under dynamic tariffs than the original flat tariff. Roldán Fernández et al. (2017) found that dynamic tariffs do reduce average electricity prices on account of reduced demand and shifted loads.

We examine how the impact of dynamic tariffs on the retail market varies with market regulation, consumer elasticities, and demand-side management (DSM) stimuli. For example, arket regulations that look to increases more competition in retail market can reduce the restrict revenue, whereas DSM stimuli (such as in-house displays and changing from bi-monthly to monthly bills) have the potential to reduce consumer demand.

In the presence of forecast errors on households electricity demand, the retailer has to participate in the balancing market. We compare the costs of these errors under different levels of forecast accuracy.

Summary of findings

Our results suggest that through the introduction of dynamic tariffs, the retailers are able to generate an additional 1% (€7.35) of annual profit from a representative household. The proposed dynamic tariff shifts about 13.8% of the peak demand to shoulder and off-peak periods, comparable with other related research (e.g. Carroll et al., 2014; Cosmo et al., 2014; Woo et al., 2014). Although the increase is small relative to the total retail profit, our analysis reveals that in 2017 the implementation of a dynamic tariff would have brought a company the size of British Gas €40 million of additional annual profit.

Our results also suggest that the retailer makes more profit if the retail market became less regulated in setting retail prices (i.e. the regulator put less restrictive constraints on the value range of retail prices). Market regulations – such as easing the licencing process to become an electricity retailer – may benefit consumers by reinforcing a more competitive retail market and transferring some of the retail profit to consumers.

Implementing DSM stimuli can reduce consumer electricity demand and further increase the retail profit. We also find that the retailer can obtain higher profit from more elastic consumers, further increasing the market gain from implementing dynamic tariffs.

Finally, our model allows us to estimate the value-added of improving the accuracy of electricity demand forecasts. Our results demonstrate that a 1% reduction in the mean absolute percentage error on the electricity demand forecast corresponds to a €0.72 increase in the annual retail profit from a representative household, or about €3.9 million/year for a major electricity supplier comparable to British Gas.

The rest of this article is structured as follows. Section 2 summarizes the literature on the impact of dynamic tariffs, and Section 3 formulates the two-stage dynamic game that models the electricity retail market. Section 4 discusses the data used herein. Section 5 specifies the input parameter values that are used in the baseline analysis, and Section 6 presents solutions derived by the model, as well as model extensions. Section 7 studies the value-added of demand forecast accuracy. Finally, Section 8 offers concluding remarks.

2. Literature review

Much of the literature on dynamic tariffs has focused on the impact on either retailer profit or consumer benefit (e.g., Doostizadeh and Ghasemi, 2012; Dagoumas and Polemis, 2017; Nezamoddini and Wang, 2017; Nilsson et al., 2018). This article looks at the electricity retail market as a whole, and analyses the effect of dynamic tariffs on retailers, consumers, as well as the entire retail market.

As downstream competition in the retail market is a necessary component for creating competition in the upstream wholesale market (Littlechild et al., 2000), research on electricity retail markets primarily focuses on market competition following the electricity industry transformations worldwide over the last three decades (Sioshansi, 2013). In their cross-country monitoring report, European electricity markets regulators ACER/CEER (2014) concluded that many European retail markets remain concentrated with three largest electricity retailers occupying over 70% of market share. The UK regulator Competition and Markets Authority (CMA) reached similar conclusions about the retail electricity market in Great Britain (CMA, 2016).

One reason for the lack of competition in the retail market is that households are constrained by search and switching costs, which refer to the time and effort needed to compare competing offers and switching between retailers. Wilson (2012) examined the effect of both costs on consumer behaviour, competition, and welfare, and argued that searching generally requires more effort and time than switching. Giulietti et al. (2014), in studying consumer search and pricing behaviour in the British domestic electricity market, found households' search costs to be relatively high. Mountain and Burns (2021), on the other hand, focused on the "loyalty tax" which is a premium charged by electricity retailers to discriminate loyal consumers. They concluded that the loyalty tax is smaller than widely considered, and that middle- and large-sized retailers impose lower loyalty tax than small-sized retailers.

Retail market competition can be enhanced by lowering the cost of switching among different tariff schemes, or increasing the number of available price schemes. While this can be economically efficient (Bohn et al., 1984), electricity consumers do not always welcome dynamic tariffs due to their perceived complexity and volatility (Brown et al., 2020). For this reason, households usually require some form of compensation to choose dynamic over flat tariffs (Ruokamo et al., 2019). Batalla-Bejerano et al. (2020), in reviewing several empirical papers, concluded that consumers exhibit heterogeneous engagement in DR programmes. The degree of engagement depends on "the level of household income, the energy characteristics of the home, the number and composition of the family unit, and the degree of environmental concern and attachment" (p. 11). Using load, price, and survey data from 119 large customers on dynamic tariffs, Boisvert et al. (2007) found 18% of the most elastic consumers provides 75% of the aggregated price response. They also argued that when peak prices are substantially higher than off-peak prices, "Commercial/Retail and Government/Education customers are more price responsive than others" (such as Health Care, Public Works, and Manufacturing customers).

A number of studies speak to the benefits of dynamic tariffs in terms of retail profit. For example, Nojavan et al. (2017) studied the impact of different price schemes on retail profit and suggest that RTP results in higher profit. Doostizadeh and Ghasemi (2012) demonstrated that in comparison with directly posting the day-ahead market prices to consumers, day-ahead RTP that maximize the retail profit subject to certain constraints would be more beneficial for energy retailers and consumers alike. Not all studies suggest that dynamic tariffs will boost retail profit. Dagoumas and Polemis (2017) combined a unit-commitment dispatch model and an econometric model and argue that DR will result in changes in the wholesale price. If the retailer is not able to anticipate these changes, they will be exposed to wholesale risks.

Finally, in terms of the effect of dynamic tariffs on consumers' costs and benefits, Borenstein et al. (1999) analysed the risks that industrial consumers face on account of fluctuating electricity bills. They found that for these customers, forward-purchase contracts can reduce bill volatility by more than 80%. Nezamoddini and Wang (2017) also studied industrial consumers, and they found that although savings realized by switching to dynamic tariffs are programme-dependent, in the majority of cases, consumers see real savings. Roldán Fernández et al. (2017) studied the financial benefits of RTP tariffs among Spanish domestic customers and found that dynamic tariffs reduce average electricity prices on account of reduced demand and shifted loads. In a Swedish DR field trial that covered 136 households, Nilsson et al. (2018) found that DR varies widely across household types. They also argue that financial incentives, such as dynamic tariffs, constitute the most effective strategy by which to increase demand flexibility.

3. Model

This section specifies the two-stage dynamic game we use to study dynamic tariffs. In the first stage, an electricity retailer attempts to maximize its profit via dynamic tariffs, and in the second stage, consumers respond. A rational retailer, therefore, will anticipate consumers' responses and set dynamic tariffs based upon its anticipation. In Section 3.1, we start from the retailer's objective function that aims at maximizing expected profit. In Section 3.2 we introduce the consumer surplus-maximization problem for a representative household which allows us to derive consumer demand as a function of retail prices. The demand function is then used as a constraint in the retailer's objective function. Section 3.3 introduces a number of additional constraints on the retailer's profit-maximization problem.

In the remainder of this article, unless otherwise specified, the term 'dynamic tariffs' refers to day-ahead RTP tariffs, where consumers are informed on electricity prices one day in advance. The proposed RTP tariff can vary on an hourly or even half-hourly basis, and can vary across days.

3.1. The retailer's problem

In the first stage, retailers buy electricity from the spot market[4] and then resell it to consumers. We assume that, initially, all retailers in the market offer a uniform flat tariff. We then assume a single retailer $\mathcal{A}$ decides to introduce a dynamic tariff, while other retailers continue to offer the original flat tariff.[5] The objective for retailer $\mathcal{A}$ is to maximize the expected profit with day-ahead RTP tariffs under certain market restrictions (which will be discussed later in Section 3.3).

The retailer's profit function, $\phi(\mathbf{x})$, for a given household on a given day is[6]

$$\phi(\mathbf{x}) = \sum_{t=1}^{T} \{\pi_t d_t - \pi_t^s E_t^s - \pi_t^{\uparrow} \Delta E_t^{\uparrow} + \pi_t^{\downarrow} \Delta E_t^{\downarrow}\}, \tag{1}$$

where $t = 1, \dots, T$ represents periods of the day,[7] and $\mathbf{x} = \{\pi_t, d_t, E_t^s, \Delta E_t^{\uparrow}, \Delta E_t^{\downarrow}\}$.

$\phi(\mathbf{x})$ comprises three parts. The first part, $\pi_t d_t$, denotes the revenue from the retail market, with π_t denoting the retail price and d_t the consumer demand. The second part, $\pi_t^s E_t^s$, denotes the cost of purchasing electricity from the day-ahead spot market, with π_t^s denoting (day-ahead) spot market prices (SMPS) and E_t^s denoting energy contracted on the day-ahead market. The third part, $\pi_t^{\uparrow} \Delta E_t^{\uparrow} - \pi_t^{\downarrow} \Delta E_t^{\downarrow}$, evaluates losses from the real-time balancing market: $\pi_t^{\uparrow}$ and $\pi_t^{\downarrow}$ respectively denote the balancing prices for the purchasing and selling of electricity; $\Delta E_t^{\uparrow}$ and $\Delta E_t^{\downarrow}$ denote the amount of energy purchased and sold on the balancing market.[8] We assume that Retailer $\mathcal{A}$ has no market power to alter wholesale prices, and so π_t, $\pi_t^{\uparrow}$, $\pi_t^{\downarrow}$ are all exogenous.

The market-balancing condition

$$d_t = E_t^s + \Delta E_t^{\uparrow} - \Delta E_t^{\downarrow}, \quad \forall t, \tag{2}$$

guarantees that supply meets demand. We impose the constraint

$$\pi_t^{\uparrow} > \pi_t^s > \pi_t^{\downarrow}, \quad \forall t,$$

which says that in any time period t, the purchase (sell) price in the real-time market must be higher (lower) than the day-ahead market prices. Given this constraint, the retailer will avoid trading on the real-time market, as doing so will always result in a profit loss. As such, the expected electricity trading at the real-time balancing market would be zero, or $\mathbb{E}[\Delta E_t^{\uparrow}] = \mathbb{E}[\Delta E_t^{\downarrow}] = 0$.

Taking the expectation of $\phi(\mathbf{x})$ in (1), the expected day-ahead profit from the retailer takes the form

$$\mathbb{E}[\phi(\mathbf{x})] = \sum_{t=1}^{T} \{\pi_t \mathbb{E}[d_t] - \pi_t^s E_t^s\}, \tag{3}$$

with the market-balancing constraint

$$\mathbb{E}[d_t] = E_t^s. \tag{4}$$

In reality, the retailer does not know what the actual demand will be on the next day, and decides the amount to bid in the day-ahead market based on demand forecasts.[9] In our empirical analysis, we assume the retailer can perfectly forecast the consumer demand — except in Section 7, where we examine the impact of forecast accuracy on retail profit.

3.2. The consumer's problem

This section sets out the second-stage consumer's surplus-maximization problem for a representative household, and derives consumer demand as a function of retailer prices. We start by assuming there are neither cross-price effects nor DSM stimuli.

[4] A spot market constitutes a day-ahead and real-time balancing market. The day-ahead price is the spot market price, and the real-time price is the balancing price.

[5] This assumption ensures consumers have the option to switch between dynamic and flat tariffs.

[6] Our modelling of the retailer's profit maximization problem extends the work of Zugno et al. (2013).

[7] t represents time intervals within a day, such as a half-hour, or peak/off-peak period.

[8] Note that wholesale power exchange can take place using forward contracts to real time, and there could be several intraday markets between day-ahead and real-time markets. However, in this paper, we simplify the wholesale market such that it consists only of a day-ahead spot market (considered to be the most liquid) and a real-time balancing market, which ensures the balancing of supply and demand.

[9] The retailer need only forecast the aggregate/average demand rather than individual-level demand.

By definition, the consumer surplus S_t for a representative household is the difference between the household benefit (from consuming electricity) and electricity bill

$$S_t = B(d_t) - d_t\pi_t, \tag{5}$$

where d_t denotes the amount of electricity consumed in period t, π_t denotes retail price in the period t, and $B(d_t)$ denotes the consumer's benefit.

Households aim to maximize the consumer surplus. Therefore, taking the first-order condition of S_t with respect to d_t and setting it to 0, we find

$$\frac{\partial S_t}{\partial d_t} = \frac{\partial B(d_t)}{\partial d_t} - \frac{\partial d_t\pi_t}{\partial d_t} = 0. \tag{6}$$

This implies

$$\frac{\partial B(d_t)}{\partial d_t} = \pi_t. \tag{7}$$

such that at the optimum the marginal benefit from consuming electricity equals the retail price.

We assume the consumer benefit, $B(d_t)$, is a quadratic function of d_t.[10] Following Schweppe et al. (2013), $B(d_t)$ can be written as

$$B(d_t) = B_{0,t} + \pi_0(d_t - d_{0,t})\left(1 + \frac{d_t - d_{0,t}}{2\epsilon_{t,t}\cdot d_{0,t}}\right), \tag{8}$$

where π_0 is the flat tariff, and $B_{0,t}$ is the consumer benefit when facing the flat tariff. $\epsilon_{t,t}$, given by $\pi_0/d_{0,t}\cdot\partial d_t/\partial\pi_t$ denotes consumers' own-price elasticity of demand. $d_{0,t}$ is the demand under the flat tariff.[11] When the consumer faces the flat tariff, $d_t = d_{0,t}$ and $B(d_t) = B_{0,t}$.

Taking the first-order condition of $B(d_t)$ in (8) and substituting it into (7), the consumer demand is

$$d_t = d_{0,t}\left(1 + \epsilon_{t,t}\frac{\pi_t - \pi_0}{\pi_0}\right). \tag{9}$$

If we now extend (9) to include cross-price effects, namely the effect of price in period t' on demand in period t, the demand function can be written as

$$d_t = d_{0,t}\left(1 + \epsilon_{t,t}\frac{\pi_t - \pi_0}{\pi_0} + \sum_{t'=1}^{T}\epsilon_{t,t'}\frac{\pi_{t'} - \pi_0}{\pi_0}\right) \tag{10}$$

where $\epsilon_{t,t'}$ is the cross-price elasticity which denotes the percentage change in demand at period t in response to the percentage change in price at period t'.

Recall that the retailer buys electricity from the day-ahead wholesale market based on their forecasts of the next day's consumer demand, namely the expected day-ahead demand (for the representative household). In expectation form, Eq. (10) can be written as

$$\mathbb{E}[d_t] = \mathbb{E}[d_{0,t}]\left(1 + \epsilon_{t,t}\frac{\pi_t - \pi_0}{\pi_0} + \sum_{t'=1}^{T}\epsilon_{t,t'}\frac{\pi_{t'} - \pi_0}{\pi_0}\right). \quad \forall t. \tag{11}$$

From (11), the expected demand comprises three components. The first component, $\mathbb{E}[d_{0,t}]$, is the expected electricity day-ahead demand under the flat tariff; the second component captures the own-price effect; and the third component captures cross-price effects.

We can further extend (11) to account for the impact of DSM stimuli. We consider two types of stimuli: a financial stimulus and an incentive stimulus. A financial stimulus offers financial rewards such as coupons to stimulate (peak) load shifting; an incentive stimulus refers to sending the customer more frequent energy bills and installing more advanced electricity monitors.

In a world where both DSM stimulus are implemented, the expected demand becomes

$$\mathbb{E}[d_t] = \mathbb{E}[d_{0,t}]\left(1 + \epsilon_{t,t}\frac{\pi_t - \pi_0 + r_t}{\pi_0} + \sum_{t'=1}^{T}\epsilon_{t,t'}\frac{\pi_{t'} - \pi_0 + r'_t}{\pi_0}\right)\cdot\kappa_t \quad ,\forall t. \tag{12}$$

where κ_t denotes the discount on demand when an incentive stimulus is implemented, which should be less than 1 in most periods.[12] r_t, on the other hand, represents the financial penalty for each unit of electricity consumed.

In (12), we assume that an incentive stimulus would proportionally change the consumer demand, hence κ_t acts as a multiplier. We also assume the financial stimuli to be additive to households' surplus function. Subtracting the financial penalty from (5) gives the households' consumer surplus function with financial incentives, or $S_t = B(d_t) - d_t\pi_t - d_t r_t$. Then, by repeating process (5)–(11), we obtain (12).

3.3. Modelling the retail market

In Sections 3.1–3.2, we introduced the first-stage retailer's profit maximization problem subject to the market-balancing constraint (4) and the second-stage demand function for a representative household (12).

In addition to (4) and (12), we place a restriction on the expected daily demand, as shown in Eq. (13).

$$D^{\text{Min}} \le \sum_{t=1}^{T}\mathbb{E}[d_t] \le D^{\text{Max}}. \tag{13}$$

The upper bound insures the system from power outages, and the lower bound ensures that the basic electricity needs of the representative household are met.

Many European countries have regulated retail tariffs. AF-Mercados et al. (2015) summarizes five principals of EU retail tariff regulation. First, retail tariffs should be transparent such that fixed and variable tariff components should be clearly stated in the consumer bill. Second, there should be no discrimination, meaning that all users under the same category and demanding the same network service should be charged the same. Third, the tariff should be easy to understand. Fourth, the tariff should be easy to predict and stable. Fifth, retail tariffs should reflect concerns about equity, such that low-income consumers should be paying no more than other consumers.

Our model specification is informed by the above five principals, in the following sense. First, households' electricity bill only comes from the dynamic tariff, which guarantees transparency (i.e. the first principal). We also assume that all households face the same dynamic rate for a certain period of time, hence the second principal is satisfied.

To incorporate the third and fourth principals, we impose an additional constraint on retail prices, as follows

$$\pi_t^{\text{Min}} \le \pi_t \le \pi_t^{\text{Max}} \quad ,\forall t. \tag{14}$$

This informs the consumer that retail prices are bounded hence avoids extreme pricing.

Under the fifth principal, we assume low-income consumers face the same dynamic tariffs as other consumers, but are given financial subsidies. We also assume the financial subsidies do not affect electricity demand, hence can be excluded from the model.

Households usually require some form of compensation to choose dynamic over flat tariffs (Ruokamo et al., 2019). Therefore, we also introduce a constraint which restricts the expected daily bill under the dynamic tariff to be no greater than a proportion (δ) of the expected daily bill under the flat tariff. We write this as

$$\sum_{t=1}^{T}\mathbb{E}[d_t]\pi_t \le \delta\cdot\sum_{t=1}^{T}\mathbb{E}[d_{0,t}]\pi_0 \tag{15}$$

(15) has three different economic interpretations. First, δ may represent consumers' risk aversion towards fluctuations in electricity prices. A small value of δ then reflects a high level of risk aversion. Second, δ

[10] Function (8) takes a quadratic form given that it is differentiable.

[11] Note that the initial price need not be a flat tariff.

[12] For some periods κ_t can exceed 1, in that the incentive stimulus increases electricity consumption. However, we would expect an effective incentive stimulus to lower total demand.

can be interpreted as the level of competition in the electricity retail market: a small value of δ indicates less retail revenue and profit, and hence a more competitive market. Third, δ may represent the extent of regulation in the retail electricity market, with a small δ indicating that the market is being heavily regulated and retailers are restricted from obtaining high revenues.[13] It is noteworthy that all three interpretations operate in the same direction, in the sense that a smaller value of δ indicates a tougher business environment (in terms of profit) for the retailer.

In summary, our model maximizes the retailer's objective function (3) subject to market constraints (4), (14), and (15), along with demand constraints (12) and (13). It can be written as

$$\begin{aligned}
\max_{\mathbf{x}} \quad & \mathbb{E}[\phi(\mathbf{x})] = \sum_{t=1}^{T}\{\pi_t \mathbb{E}[d_t] - \pi_t^s E_t^s\}, \\
\text{subject to} \quad & \mathbb{E}[d_t] = E_t^s, \quad \forall t \\
& \mathbb{E}[d_t] = \mathbb{E}[d_{0,t}]\left(1 + \epsilon_{t,t}\frac{\pi_t - \pi_0 + r_t}{\pi_0} + \sum_{t'=1}^{T}\epsilon_{t,t'}\frac{\pi_{t'} - \pi_{0,} + r_t'}{\pi_0}\right)\cdot \kappa_t, \quad \forall t \\
& D^{\mathrm{Min}} \le \sum_{t=1}^{T}\mathbb{E}[d_t] \le D^{\mathrm{Max}} \\
& \pi_t^{\mathrm{Min}} \le \pi_t \le \pi_t^{\mathrm{Max}}, \quad \forall t \\
& \sum_{t=1}^{T}\mathbb{E}[d_t]\pi_t \le \delta \cdot \sum_{t=1}^{T}\mathbb{E}[d_{0,t}]\pi_0.
\end{aligned} \tag{16}$$

where as noted $\mathbf{x} = \{\pi_t, d_t, E_t^s, \Delta E_t^{\uparrow}, \Delta E_t^{\downarrow}\}$. We solve the profit maximization problem using the barrier method, which formulates the inequality constrained problem as an equality constrained problem, such that Newton's method can be applied (see Appendix for details). To solve the profit maximization problem, we use consumer demand under a flat tariff ($d_{0,t}$), elasticities ($\epsilon_{t,t}$ and $\epsilon_{t,t'}$), and day-ahead SMPs (π_t^s) as input data. We then examine how the dynamic prices that maximize retail profit impact consumer demand, bills, as well as market gain.

4. Data

This section describes the datasets used as inputs into the two-stage dynamic game. Consumer demand as well as the upper and lower bounds of retail prices, come from the Ireland Electricity Smart Metering Trials (IESMT) programme. The tariff structure of the IESMT programme is used as model inputs, specifically providing maximum and minimum values on dynamic tariffs charged by the retailer.[14],[15]

Table 1
TOU tariff schemes.

	Prices by Tariff Scheme (Euro Cents/kWh)					
	A	B	C	D	W	Control
Weekday Off-peak 23.00–08.00	12.00	11.00	10.00	9.00	10.00	14.10
Weekday Shoulder 08.00–17.00 & 19.00–23.00	14.00	13.50	13.00	12.50	14.00	14.10
Weekday/Holiday Peak 17.00–19.00	20.00	26.00	32.00	38.00	38.00	14.10
Weekend/Holiday Off-peak 23.00–08.00	12.00	11.00	10.00	9.00	10.00	14.10
Weekend Shoulder 08.00–23.00	14.00	13.50	13.00	12.50	10.00	14.10
No. of Households	1010	382	1018	373	74	782

Table 2
A summary of Irish SMPs (Euro Cents/kWh), 2010.
Source: http://www.sem-o.com/.

	Off-peak	Shoulder	Peak
Mean	3.84	5.70	8.47
Minimum	2.39	3.61	3.18
Maximum	6.53	13.11	30.00

Table 1 summarizes the tariff schemes, and the number of participants (i.e. households) with complete records. The trial consisted of six different types of tariff schemes: four weekday TOU tariff schemes (A-D), a weekend tariff scheme (W), and one flat tariff scheme (control). Given the design of the trial, among the four TOU tariff schemes, a high peak price always comes with low shoulder and off-peak prices.

Households who face TOU tariffs (Plans A-D) are randomly allocated in terms of the following four incentive DSM stimuli (CER, 2012): a bi-monthly electricity bill with a detailed energy statement; a monthly electricity bill with a detailed energy statement; a bi-monthly electricity bill with a detailed energy statement and an electricity monitor; or a bi-monthly electricity bill with a detailed energy statement and an overall load reduction incentive.

In addition to the IESMT dataset, we also collect the half-hourly day-ahead SMP data from the Single Electricity Market Operator (SEMO). We then aggregate the half-hourly data such that there are three prices representing the off-peak, shoulder, and peak SMPs. The summary statistics for SMPs are listed in Table 2. Not surprisingly, the average peak SMP is much higher than the average shoulder and off-peak SMP.

5. Baseline settings

Table 3 presents the default values for the input parameters used in the baseline model. $\kappa_t = 1$ imposes the constraint that the DSM stimuli have no impact on consumer demand for off-peak (O), shoulder (S), and peak (P) periods. We set $\delta = 1$ in the incentive constraint (15), such that the representative household is risk-neutral and will prefer the dynamic tariff as long as it incurs a lower bill (than the flat tariff). Day-ahead SMPs (π_t^s) are obtained from the SEMO introduced in Section 4, and the flat tariff (π_0) is taken from the IESMT programme 1.

The demand for the representative household under the flat tariff ($d_{0,t}$) is set using the average demand among households under the flat tariff (i.e. the 'Control' group in Table 1). We assume that when facing dynamic tariffs for a given day, the representative household would consume no more (less) than 105% (95%) of the total demand

[13] An alternative constraint is $\sum_{t=1}^{T}\mathbb{E}[d_t]\pi_t \le \delta\cdot\sum_{t=1}^{T}\mathbb{E}[d_t]\pi_0$. If $\delta = 1$ this ensures the household's expected bill under the dynamic tariff is no more than the expected bill if they consume the same demand under the dynamic tariff but face the flat rate. The difference is that Eq. (15) is an *ex-ante* constraint while the alternative is an *ex-post* constraint. (15) assumes the household would re-optimize its consumption pattern when facing the dynamic tariff, whereas the latter one compares dynamic with flat tariffs holding demand constant. The reason we choose the *ex-ante* constraint (15) is that it gives δ three different interpretations, while if we use the *ex-post* constraint, the interpretation of market competitiveness does not hold. The reason is that the right-hand-side formula no longer represent consumer's outside option of switching back to the original flat tariff.

[14] The IESMT programme is one of the largest smart metering programme in Europe at the time when it was released. Smart metres were installed to record households' half-hourly electricity demand between 14 July 2009 and 31 December 2010, with 3,639 households participating in and completing the trial. Before 2010, all households were offered the standard Electric Ireland flat tariff of 14.1 euro cents/kWh; then, during the entire year of 2010, most households were been randomly allocated with four different types of TOU tariffs, the rest still pays the original flat tariff.

[15] The impact of the Irish trial (in particular the TOU tariffs and DSM stimuli) on households' electricity demand is well documented (e.g. Cosmo et al., 2014; Kiguchi et al., 2019; O'Neill and Weeks, 2018). It is noteworthy that instead of estimating the treatment effects attributed to the introduction of TOU tariffs, we use the IESMT data as inputs to our model which solves the retailer's profit maximization problem.

Table 3
Baseline setting: Values of input parameters.

κ_t	=1
δ	=1
π_t^s	day-ahead SMPS collected from SEMO
π_0	=14.1 euro cents/kWh, the flat tariff from the IESMT programme
$d_{0,t}$	The representative household electricity demand under flat tariff in 2010 from the IESMT programme
D^{Min}	$0.95 \times \sum_{t=1}^{T} \mathbb{E}[d_{0,t}]$ (euro cents/kWh)
D^{Max}	$1.05 \times \sum_{t=1}^{T} \mathbb{E}[d_{0,t}]$ (euro cents/kWh)
π_t^{Min}	$\pi_O^{\text{Min}} = 9,\ \pi_S^{\text{Min}} = 12.5,\ \pi_P^{\text{Min}} = 20$
π_t^{Max}	$\pi_O^{\text{Max}} = 12,\ \pi_S^{\text{Max}} = 14,\ \pi_P^{\text{Max}} = 38$
$\epsilon_{t,t}$, $\epsilon_{t,t'}$:	Table 4

Table 4
Elasticities of demand.

	t'		
	O	S	P
$\epsilon_{O,t'}$	−0.039	0.025	0.014
$\epsilon_{S,t'}$	0.031	−0.067	0.036
$\epsilon_{P,t'}$	0.044	0.090	−0.134

when facing the flat tariff (i.e. $D^{\text{Min}} = 0.95 \times \sum_{t=1}^{T} \mathbb{E}[d_{0,t}]$ and $D^{\text{Max}} = 1.05 \times \sum_{t=1}^{T} \mathbb{E}[d_{0,t}]$).[16]

The market regulator sets the upper and lower bounds of retail prices (denoted as π_t^{Min} and π_t^{Max}, respectively). At the time of writing the Irish market regulator has not yet published such guidelines. In our baseline settings, the upper and lower bounds are taken from the TOU tariffs from the IESMT. For example, the range for off-peak TOU prices is set within the interval of $[9, 12]$ (see Table 1).

In terms of consumers' price elasticities of demand, we assume that elasticities for all households are identical.[17] We follow Mountain and Lawson (1992) who estimated consumer elasticities for different periods of the day in Ontario, Canada.[18] The elasticities for the baseline scenario are listed in Table 4,[19] where $\epsilon_{t,t'}$ represents the impact prices in period t on the electricity demand in period t'. One may argue that with the installation of more advanced smart metres and smart household appliances and the increased use of electric vehicles, households may become more price sensitive. Therefore, in Section 6.3 we vary elasticities and investigate its impact on the retail electricity market.

6. Results

This section examines the dynamic tariffs that maximize retail profit, and the associated impact on consumer demand, consumer bill, retail profit, and market gain. After obtaining the dynamic retail tariffs that solve the retailer's problem for each day, we analyse the results for a single year. As the electricity retailer sets the day ahead retail prices based upon the wholesale price (specifically, the day-ahead spot market price), the retailer's profit-maximization problem can be solved for each single day.

In Section 6.1 we analyse the baseline results. Section 6.2 examines the impact of relaxing the retail price constraints, and in Section 6.3 we analyse the influence of price elasticities on the retail market. Section 6.4 investigates the impact of market regulation, and Section 6.5 evaluates the effects of DSM stimuli. Comparing the results of these changes with the baseline enables us to provide information to retailers and policymakers in terms of the potential impact of dynamic tariffs and demand response policies on the retail electricity market.

Unless otherwise specified, in the remainder of this paper peak prices, shoulder prices, and off-peak prices respectively refer to peak retail prices, shoulder retail prices, and off-peak retail prices, all under dynamic tariffs.

6.1. Baseline results

Using the data introduced in Section 4 as inputs to the two-stage dynamic game (16), we obtain dynamic retail prices that maximizes the retailer's profit. We then examine the relationship between the dynamic retail prices and the associated SMPS. Finally, we compare the retail profit, consumer demand, consumer bill, and market gain that arise from the dynamic tariffs to those from the original flat tariff.

The distribution of dynamic retail prices

Table 5 categorizes solved dynamic retail prices into four different cases based upon the relationship between peak, shoulder and off-peak prices. Recall that the retailer maximizes its profit subject to the consumers' incentive constraint (15), which sets a ceiling to the retail revenue for a given day.[20] As a result, if the retailer wishes to set a high price for one period, the prices for other periods need to be lower, such that the total bill will not exceed the ceiling. As an example, days with high peak prices correspond to low shoulder and off-peak prices.

Cases a.1–a.4 are ranked in descending order of peak SMPS. Case a.1 represents days with the highest average peak SMPS (13.39 euro cents/kWh) and correspondingly high peak retail price. Due to the incentive constraint (15), the shoulder and off-peak prices are equal to their lower bounds (superscript L) of 9 euro cents/kWh and 12.5 euro cents/kWh, respectively. The average peak SMPS for Case a.2 are lower than those in Case a.1. The attendant lower peak retail price implies that to maximize profit under the incentive constraint (15), the retailer is able to set shoulder prices at the upper bound (represented by the superscript U) of 14 euro cents/kWh. The off-peak prices are equal to the lower bound of 9 euro cents/kWh.

The average peak SMPS for Case a.3 are the second-lowest among all cases, resulting in relatively low peak retail prices. Again, lower peak retail prices has the effect that shoulder prices are set equal to the upper bound of 14 euro cents/kWh, and the off-peak prices are above the lower bound of 9 euro cents/kWh.

Case a.4 has the lowest average peak SMP among all cases. In this instance we observe peak retail price set equal to the lower bound of 20 euro cents/kWh. As with Case a.3, to maximize profit the retailer sets

[16] We assume that under dynamic tariffs, for any given day the total demand for the representative household cannot exceed +/− 5% of the total demand under the flat tariff. Changing +/− 5%, for example, to +/− 10% will have negligible impact on the results. The main reason is that the price constraint (14) ensures retail price will neither be too high nor too low — since households are inelastic, their electricity demand will not substantially be affected by dynamic tariffs, hence the (total) demand will not change substantially under dynamic tariffs relative to the flat tariff.

[17] In Section 8 we discuss the challenges inherent in modelling scenarios when elasticities are not identical.

[18] We are unable to provide reliable estimates of elasticities from the IESMT programme. Given the design of the trial, shoulder and off-peak prices suffer from high collinearity. One solution might be to exclude either shoulder or off-peak prices from the regression, but doing so would result in omitted-variable bias. Attempts have also been made to find appropriate regression methods by which to identify elasticities (e.g. the restricted LASSO), but there has been limited success in this area.

[19] The values are taken from Mountain and Lawson (1992), page 196, Table 5 ("Rate Cell 16 Winter").

[20] Recall that the incentive constraint (15):

$$\sum_{t=1}^{T} \mathbb{E}[d_t]\pi_t \le \delta \cdot \sum_{t=1}^{T} \mathbb{E}[d_{0,t}]\pi_0$$

requires that the consumer bill under the dynamic tariff be no more than that under the flat tariff. Since the expected demand under the flat tariff $\mathbb{E}[d_{0,t}]$ and the flat price π_0 are both given, the upper limit for the consumer bill is given as a constant.

Table 5
Distribution of the dynamic retail prices (euro cents/kWh)- Baseline.

Case	Dynamic Retail Prices[a]			Average Spot Prices			No. of days
	Off-peak	Shoulder	Peak	Off-peak	Shoulder	Peak	
a.1	9^L	12.5^L	> 29	4.13	5.50	*13.39*	119
a.2	9^L	14^U	$(20^L,29)$	3.68	5.96	*7.07*	127
a.3	$(9^L,12^U)$	14^U	$(20^L,26)$	3.59	5.55	*5.41*	25
a.4	$(9^L,12^U)$	14^U	20^L	3.74	5.63	*4.93*	94

[a]The superscript L and U represent the lower and upper bounds, respectively.

Table 6
Flat tariff *vs.* dynamic tariff, annual comparison.

	Retail Profit (€)	Bill (€)	Demand (kWh)	Market Gain (€)
Flat Tariff	711.26	1199.34	8506	–
Dynamic Tariff	718.61	1199.34	8526	7.35

the shoulder price equal the upper bound of 14 euro cents/kWh, and the off-peak retail prices exceed the lower bound of 9 euro cents/kWh in order that the incentive constraint (15) binds.

In summary, Table 5 demonstrates the relationship between peak SMP and the dynamic retail prices that maximizes the retailer's profit. When the peak SMP is high, setting a *high peak* retail price becomes the retailer's priority; while when the peak SMP is relatively low (cases a.2–a.4), setting a *high shoulder* retail price becomes the retailer's priority. However, setting the shoulder retail price to its upper bound is not sufficient to bind (15).[21] Hence to maximize profit, the retailer needs to increase retail prices in other periods (peak and off-peak) until the constraint binds.

Comparing dynamic with flat tariffs

Table 6 compares the impact of dynamic and flat tariffs using the following measures: the average annual profit for the retailer from the representative household (retail profit); the average annual electricity bill for the representative household (bill); the average annual electricity demand for the representative household (demand); and the average annual monetary gains derived from the dynamic tariff, relative to the flat tariff (market gain).[22]

When the incentive constraint (15) is binds, households pay the same bill under flat and dynamic tariffs such that all market gain comes from the increase in the retail profit.[23] From Table 6 under the dynamic tariff the retailer earns an additional €7.35 of annual profit from a representative household. Although the magnitude is small relative to the annual profit (1%),[24][25] if all domestic customers of British Gas were to switch to the dynamic tariff, annual profit would increase by approximately €40 million.

Table 7
Demand reallocation in the baseline settings (kWh/Year).

	Demand		Change	
	Flat	Dynamic	Level Change	% Change
Off-peak	1852	1895	43	2.4%
Shoulder	5519	5652	133	2.4%
Peak	1135	978	−157	−13.8%

We also note that under dynamic tariffs consumer demand increases by 20 kWh/year. However, this does not necessarily mean that the household's welfare is increasing under dynamic tariffs, given that the increase in electricity demand comes with peak load shifting, and households may obtain different utilities in different periods.

Table 7 compares the annual consumer demand (for the representative household) under dynamic tariff with flat tariffs, for off-peak, shoulder, and peak periods. Overall, dynamic tariffs result in a 13.8% shift of demand from peak to other periods.[26] Relative to the baseline scenario, the dynamic tariff reduces the representative household's peak demand by 156.2 kWh/year/household. 133.4 kWh (or 85%) of this is reallocated to shoulder periods and 42.9 kWh (or 27%) is reallocated to off-peak periods. This finding is comparable to a number of other studies (e.g. Carroll et al., 2014; Cosmo et al., 2014; Woo et al., 2014).

In summary, under the baseline settings we find that peak SMPs are a critical factor in determining dynamic retail prices. Given the incentive constraint (15) that restricts the retailer's total revenue, a high peak retail price is associated with low shoulder and off-peak prices, and vice versa. We also find that the dynamic tariffs are beneficial in terms of substantially increasing retailer profit and shifting and reducing peak load. Although the retailer gets all the market gain, Section 6.4 demonstrates that market regulation can redistribute these gains, resulting in a lower electricity bill for the representative household.

6.2. The role of price constraints

In the baseline scenario the retail-price constraints are based upon tariffs used in the Irish trial (recall Table 3). One might argue that the available ranges for the retailer to set retail prices are too small, such that the advantage of dynamic tariffs cannot be fully exploited. In this section we widen the range between the lower and upper bounds of the retail prices, allowing the retailer increased flexibility in setting retail prices. Relative to the baseline scenario, we reduce (increase) the lower (upper) bound by 2 euro cents, namely

$$\pi_O^{\text{Min}} = 7 \text{ euro cents/kWh} \quad \text{and} \quad \pi_O^{\text{Max}} = 14 \text{ euro cents/kWh}; \tag{17}$$

$$\pi_S^{\text{Min}} = 10.5 \text{ euro cents/kWh} \quad \text{and} \quad \pi_S^{\text{Max}} = 16 \text{ euro cents/kWh}; \tag{18}$$

$$\pi_P^{\text{Min}} = 18 \text{ euro cents/kWh} \quad \text{and} \quad \pi_P^{\text{Max}} = 40 \text{ euro cents/kWh}. \tag{19}$$

Given that his allows the retailer to set higher (or lower) prices in the expectation of additional profit, we would expect to observe an increase in the retail profit under constraints (17)–(19) relative to the baseline scenario.

Table 8 presents the distribution of dynamic retail prices that maximizes the retail profit under the price constraints (17)–(19). Similar to the baseline scenario, we categorize the retail prices into three different cases based upon average peak SMPS. Days that are categorized into

[21] In the baseline scenario the upper bound for shoulder retail prices is lower than the flat tariff (See Table 3).

[22] This is defined as the additional profit earned by the retailer plus the reduction in the household's electricity bill. Market gain for a single day can be expressed as

$$\text{Market gain} = \sum_{t=1}^{T} \{ \left[d_t(\pi_t - \pi_t^s) - d_{0,t}(\pi_0 - \pi_t^s) \right] + (d_{0,t}\pi_0 - d_t\pi_t) \},$$

where the first term measures the change in the retail profit and the second term represents the change in the consumer bill.

[23] In Section 6.4 we show that, despite this, the implementation of less restrictive market regulation rules (such as easing the licencing process to become an electricity retailer) can redistribute market gain.

[24] Note that other costs such as network costs and operating costs, are ignored in the retailer's profit function. Once those costs are taken into account, the number exceeds 1%.

[25] As an example, in 2017 the total number of UK households was 27.2 million and the market share for an electricity supplier such as British Gas was 20%. See Office for National Statistics and Ofgem.

[26] In 2010 both Britain and Ireland experienced the coldest winter on record, resulting in households' electricity consumption being substantially higher than other years.

Table 8
Distribution of dynamic retail prices (euro cents/kWh).

Case	Dynamic Retail Prices[a]			Average Spot Prices			No. of Days
	Off-peak	Shoulder	Peak	Off-peak	Shoulder	Peak	
b.1	7^L	$(10.5^L, 16^U)$	40^U	4.13	5.50	*13.39*	119
b.2	$[7^L, 7.4)$	16^U	$(18^L,21)$	3.67	6.05	*6.86*	121
b.3	$[7^L, 8.2)$	$(15.4, 16^U]$	18^L	3.73	5.54	*5.33*	125

[a]The superscript L and U represent the lower and upper bounds, respectively.

Table 9
Comparing flat tariff with dynamic tariffs.

	Profit (€)	Bill (€)	Demand (kWh)	Market Gain (€)
Flat Tariff	711.26	1199.34	8506	–
Dynamic Tariffs				
Baseline (Table 6)	718.61	1199.34	8526	7.35
Constraints (17)–(19)	724.82	1199.34	8476	13.56

Table 10
Demand reallocation under constraints (17)–(19), kWh/Year.

	Demand		Change in Demand	
	Flat	Dynamic	Level Change	% Change
Off-peak	1852	1912	60	3.2%
Shoulder	5519	5595	76	1.4%
Peak	1135	969	−166	−14.6%

Case b.1 are the same as days that are categorized into Case a.1 (in the baseline scenario), indicating that in this instance, the setting of high peak prices on days with high SMPs is not affected by the change in price constraints.

The relaxation of the retail price constraints allows the retailer to increase the peak retail price relative to the baseline scenario, until it equals the upper bound in (19) (i.e. 40 euro cents/kWh). Given the nature of the incentive constraint, the retailer will again lower the off-peak retail price to its lower bound in (17) (i.e. 7 euro cents/kWh). The retailer will then increase the shoulder retail price until the incentive constraint (15) binds.

Case b.2 is comparable to Cases a.2-a.3 in Table 5, as the shoulder prices are equal to their upper bounds while off-peak and peak prices are low. Average peak SMPs are lower than Case b.1 but higher than Case b.3. In this case, shoulder prices equal the upper bound at 16 euro cents/kWh, while the peak prices is either equal to or greater than the lower bound (i.e. within (18,21) euro cents/kWh).

Finally, Case b.3 is comparable to Case a.4 in Table 5, with the average peak SMPs the lowest of all cases, and the peak retail price set equal to the lower bound (i.e. 18 euro cents/kWh). In this case, the shoulder retail price is either equal to or less than its upper bound (i.e. within (15.4, 16] euro cents/kWh), and the off-peak price is either equal to or greater than its lower bound (i.e. within [7, 8.2) euro cents/kWh).

Under the price constraints (17)–(19), when the shoulder retail price is lower than the upper bound, we observe off-peak retail prices equal to the lower bound. Similarly, when off-peak retail price exceed th lower bound, we observe shoulder retail prices equalling to its upper bound.

This pricing behaviour is consistent with a retailer maximizing profit under the incentive constraint (15) – while the peak retail price is equal to the lower bound, the retailer makes more profit from setting high shoulder retail prices. For days where there is low demand or low SMPs, setting the shoulder retail price less than its upper bound is sufficient to bind the constraint (15) (and thereby maximize the retail profit). For days where demand is high or high SMPs, setting the shoulder retail price to its upper bound is not sufficient to bind the constraint (15), such that the retailer would raise the off-peak retail prices above its lower bound for higher profit.

Comparing dynamic and flat tariffs under the new constraints

Table 9 presents the retail profit, consumer demand, consumer bill, and market gain from the representative household under the new constraints. Given that the new constraints are less restrictive than the baseline scenario,[27] the retail profit almost doubles from €7.35/year/household to €13.56/year/household. Due to the incentive constraint (15), the consumer bill is still the same as under the flat tariff, but the total demand is substantially reduced by 30 kWh/year (0.35%/year) relative to the total demand under the flat tariff.

Given that most electricity is consumed during shoulder (and off-peak) periods,[28] and for the baseline scenario the upper bounds for both off-peak and shoulder prices are lower than those under the flat tariff,[29] dynamic tariffs result in higher demand relative to the flat tariff. However, in the case of the new constraints (17)–(19), shoulder retail prices can be higher than the flat tariff, meaning that households would consume less if the shoulder retail price were equal to the new upper bound. Finally, similar to the baseline scenario, all the market gain belongs to the retailer.

In Table 9 we observe that the dynamic tariff does not necessarily reduce consumer demand, as consumer demand under the dynamic tariff can be either higher or lower than the flat tariff. In fact, whether dynamic tariffs reduce demand will mostly depend on whether the average price of dynamic tariffs is higher than the flat tariff. On the other hand, as will be shown in Section 6.5, DSM stimuli can be used as major tools for energy conservation.

Table 10 shows how dynamic tariffs reallocate electricity demand under the new constraints (17)–(19). Relative to flat-tariffs we observe that for the representative household, dynamic tariffs reduce peak demand by 166 kWh in 2010, equivalent to 14.6% of peak demand; this is slightly higher than the baseline scenario, at 13.8%. We also note that 46% (36%) of the reduction in peak demand shifts to the shoulder (off-peak) periods.

In summary, as the new retail-price constraints (17)–(19) give the retailer more flexibility in setting the retail prices, the retailer makes more profit, thus amplifying the market gain. In addition, we observe that the representative household reduces peak load. Again, almost the entire market gain goes to the retailer, but market regulation can redistribute the gain, as Section 6.4 shows.

6.3. The role of elasticities

In Sections 6.1 and 6.2 we examined the impact of dynamic tariffs on the retail market under the baseline scenario and more flexible price constraints. In both cases we used the price elasticities estimated by Mountain and Lawson (1992), as presented in Table 4. Given the installation of more advanced smart metres and smart household appliances together the increased use of electric vehicles, suggests that consumers, faced with stronger monetary incentives and smart technology, may become be more price-sensitive.

[27] As shown in Table 3, under the baseline scenario the range between lower and upper bounds for off-peak periods is 3 euro cents/kWh; for shoulder periods the range is 1.5 euro cents/kWh. Under the new constraints (17)–(19), the range for off-peak periods is 7 euro cents/kWh; for shoulder periods the range is 5.5 euro cents/kWh.

[28] Peak periods have the highest average hourly demand, but off-peak and shoulder periods have the highest overall demand.

[29] Recall that in the baseline scenario, the upper bound for off-peak retail price is 9 euro cents/kWh and for shoulder retail price is 12.5 euro cents/kWh; while the flat tariff is constant at 14.1 euro cents/kWh.

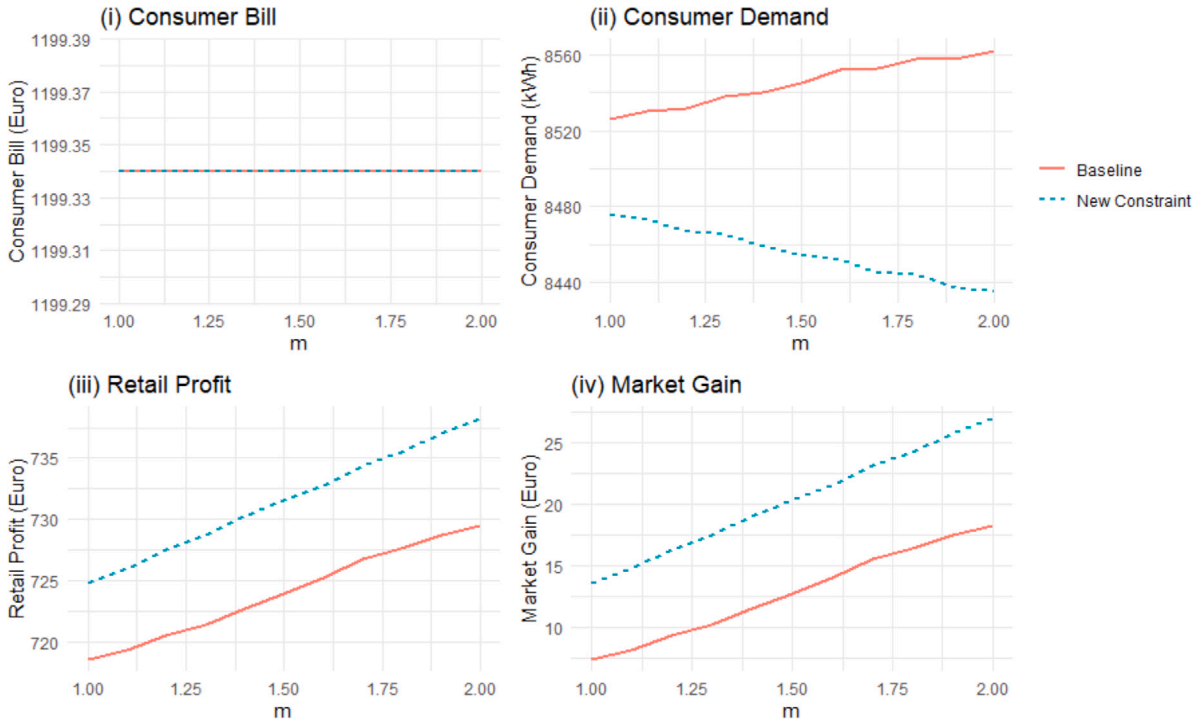

Fig. 1. Impact of m on Consumer Demand and Bill, Retail Profit, and Market Gain, Baseline *vs.* New Constraint.

Fig. 1 demonstrates the impact of adjusting elasticities by a factor $m \in (1, 2]$ on consumer demand, consumer bill, retail profit, and market gain. The solid lines (dashed lines) represent baseline scenario (new retail price constraints).

Fig. 1(i) plots the relationship between m and consumer bill. Recall that the incentive constraint (15) restricts the maximum bill that the representative household would pay under dynamic tariffs. To maximize profit, the retailer would set prices so that the constraint is binding. Put another way, the consumer bill under a dynamic tariff will be equal to the bill under the flat tariff. For both scenarios, this indicates consumer bills are constant (represented by the horizontal lines in Fig. 1(i)) regardless of consumer elasticities.

Fig. 1(ii) shows that under the baseline scenario, consumer demand increases with elasticities (i.e. increases with m). This follows since under the baseline scenario the upper bounds for off-peak and shoulder retail prices are lower than the flat tariff rate (i.e, 12 and 14 are lower than 14.1 in Table 3). As a result, as household become more price elastic, we observe an increase in consumer demand under dynamic tariffs.[30] Note that in this instance although electricity consumption is increasing under dynamic tariffs, the bill unchanged. Whether this makes the consumers better-off depends on whether the utility from enjoying more energy usage exceeds the dis-utility from their peak load shifting.

Under the new retail price constraints (17)–(19), the upper bound of the shoulder prices is higher than the flat tariff, such that demand under the dynamic tariff will be lower than that under the flat tariff. As m increases and consumer demand becomes more price elastic, we observe the reverse relative to the baselines constraints: demand falls with increasing m. In this scenario, consumers are paying the same bill as under the flat tariff, but consume less electricity, potentially resulting in a reduction in consumer's welfare. However, from policymakers' perspective, less demand means less greenhouse gas emission and greater social benefit.[31]

Fig. 1(ii) has an important policy implication. Dynamic tariffs do not necessarily reduce consumer demand relative to the flat tariff. While the peak price is important in shifting the peak demand, shoulder and off-peak prices are crucial in determining the total consumer demand. Put another way, to lower total demand (perhaps to achieve energy conservation goals), the regulator needs to allow retailers to set shoulder and off-peak prices higher than the flat tariff rate.

Fig. 1(iii) demonstrates the impact of m on retail profit. Note that the retailer makes a profit from buying low and selling high. Hence, from the retailer's perspective the role of dynamic tariffs is to shift the consumer demand from low-profit to high-profit periods. This suggests a positive relationship between m and retail profit. As Section 6.2 discusses, the more flexible constraints will allow a retailer to make more profit from households.

Finally, Fig. 1(iv) presents the relationship between m and the market gain. Recall that the market gain defined in is the sum of the increase in the retail profit and the reduction in the consumer bill. Since the retail profit increases with m while the consumer bill stays constant, the market gain fully traces out the retail profit in both scenarios.

In summary, this section has demonstrated that retail profit increases with the price elasticity of demand. This is because the retailer is able to arbitrage from more elastic households through buying low and selling high. We also find that under different retail price constraints, the relationship between consumer demand and consumer elasticities can be in opposite signs (as Fig. 1(ii) shows). As discussed in Section 6.2, the configuration of the retail-price constraints relative to the flat tariff is the key to determine whether consumer demand is upward or downward sloping with consumer elasticities.

[30] Although the peak price is higher than the flat tariff, the total peak demand is much lower than the total shoulder and off-peak demand. In this sense total demand is more sensitive to shoulder and off-peak prices.

[31] Estimating the overall impact would require estimating households' utilities from consuming electricity, the marginal emission of electricity generation, and the social cost of carbon. These matters are left to future research.

Table 11
Impact of market regulation on the retail market.

	Profit (€)		Bill (€)		Demand	Market
	Profit	Δ^{*}	Bill	Δ^{**}	(kWh)	Gain (€)
$\delta = 1$	718.61	7.35	1199.34	0.00	8526	7.35
$\delta = 0.998$	716.16	4.90	1196.94	2.40	8524	7.30
$\delta = 0.996$	713.72	2.46	1194.54	4.80	8523	7.26
$\delta = 0.994$	711.27	0.01	1192.14	7.20	8521	7.21
Flat Tariff	711.26	–	1199.34	–	8506	–

Δ^{*} Difference between dynamic and flat tariff; Δ^{**} difference between flat and dynamic tariff.

6.4. The role of market regulation

In this section we relax the assumption that the consumer bill under the dynamic tariff cannot exceed the bill under the flat tariff. Instead, we allow δ in the incentive constraint (15) to be less than 1.[32] Recall that δ has multiple interpretations, including the level of market competition, consumers' risk aversion, and market regulation, all of which increase as δ decreases.

We assume that consumer risk aversion is fixed and focus upon the impact of reducing δ to reflect a change in market regulation in accordance with the objective of increasing competition or more frequent monitoring activities.[33]

Table 11 presents the impact of market regulation on retail profit, consumer demand, consumer bill, and market gain. We reduce the value of δ until the retailer cannot make more profit from the dynamic tariff relative to the flat tariff. The table shows that the retailer would prefer the flat tariff over the dynamic tariff when $\delta < 0.994$. If there is too much regulation in the market (i.e. a small δ), this reduces the retailer's incentives to implement dynamic tariffs.

The reason is as follows. Recall that the incentive constraint (15) sets the upper bound of the consumer bill under dynamic tariffs. For the representative household, this is approximately €1200 (see Table 6). A 0.6% reduction in δ generates a €7.2 reduction in the representative household's annual bill, or about 98% of the profit that the retailer makes from implementing the dynamic tariff in the baseline scenario (€7.35/year/household).

From Table 11 we see that market regulation has little effect on consumer demand. Although market regulation slightly reduces market gain, it transfers market gain from the retailer to households. The reason is that in a competitive electricity market where the impact of CO_2 emissions from electricity generation has been accounted for through carbon pricing, there is always a trade-off between the distribution of market gains and efficiency — the regulator's intervention makes the retail market less efficient. Despite this, at some certain levels of market regulation (i.e, $\delta \in [0.994, 1)$ in our model), both the retailer and households benefit from the dynamic tariff.

6.5. The role of DSM stimuli

Allcott (2011), Gans et al. (2013) each found that demand-side management (DSM) stimuli are associated with declines in electricity consumption. In this section we separately examine the impact of two DSM stimuli: switching from monthly to bi-monthly bills and installing in-house displays, both of which were implemented as part of the IESMT programme.

[32] The value of δ can exceed 1 in the sense that consumers would pay more under the dynamic tariff than under the flat tariff. However, this violates the condition that applying dynamic tariffs should make both consumers and retailers better off.

[33] Note that the three different interpretations for δ overlap. For example, market regulation rules can result in a more competitive market.

If we introduce DSM stimuli in the form of an adjustment factor κ_t as discussed in Section 3.2, then using (12), the consumer's demand function may be written as

$$\mathbb{E}[d_t] = \mathbb{E}[d_{0,t}]\left(1 + \epsilon_{t,t}\frac{\pi_t - \pi_0}{\pi_0} + \sum_{t'} \epsilon_{t,t'}\frac{\pi_{t'} - \pi_0}{\pi_0}\right) \cdot \kappa_t \quad, \forall t \in \{O, S, P\}.$$

Recall that κ_t denotes the discount on demand when an incentive stimulus is implemented. Using the IESMT dataset, Cosmo et al. (2014) found that relative to bi-monthly bills, monthly bills under TOU schemes reduce peak demand by 1.3%, shoulder demand by 0.2%, and off-peak demand by 1.6%. The installation of in-house displays reduce peak demand by 1.0%, shoulder demand by 0.0%, and off-peak demand by 1.2%.[34] Assuming that households react similarly to incentive stimuli under the dynamic tariff and the TOU schemes, we use Cosmo et al. (2014)'s results as estimates of κ_t ($t \in \{O, S, P\}$) when switching from bi-monthly to monthly bills, or when installing an in-house display under bi-monthly bills.

Table 12 shows the impact of introducing monthly billing and in-house displays on retail profit, consumer demand, consumer bill, and market gain for the representative household. Although both TOU stimuli lower consumer demand, given the incentive constraint (15), the consumer bill remains unchanged. Put another way, if we focus on the *ceteris paribus* effect of monthly billing and in-house displays, we find that these incentive stimuli would benefit the retailer with higher profit, but harms the household by facing the same bill for lower demand.

To transfer the extra market gain (from introducing monthly billing and in-house displays) from the retailer to households, the regulator needs to increase the level of market competition, so as to reduce δ in Eq. (15). The mechanism behind this was discussed in Section 6.4. The effect of introducing monthly billing and in-house displays in conjunction with a lower δ is also demonstrated in Table 12, where the last two rows show the results where the incentive stimuli interact with market regulation (by setting $\delta = 0.995$ according to Table 11). It is suggested that under the dynamic tariff scheme, the incentive stimuli in conjunction with market regulation will result in both retailer and households benefiting from dynamic tariffs — households are facing lower bills and retailers is making higher profit (than the flat tariff).

7. Forecast errors

Every year electricity retailers invest substantially into improving forecast accuracy of the day-ahead load. In a world with high renewable penetration, the real-time balancing prices becomes more volatile, further boosting the monetary gain from improving forecast accuracy. In this section we estimate the monetary value of accurate forecasts of day-ahead electricity demand.

Recall the total profit for a retailer is the profit from the day-ahead market minus the loss from trading in the real-time balancing market:

$$\phi(\mathbf{x}) = \sum_{t=1}^{T}\{\pi_t d_t - \pi_t^s E_t^s - \pi_t^{\uparrow}\Delta E_t^{\uparrow} + \pi_t^{\downarrow}\Delta E_t^{\downarrow}\}, \tag{1}$$

which is restricted by the market-balancing constraint

$$d_t - E_t^s = \Delta E_t^{\uparrow} - \Delta E_t^{\downarrow} \quad \forall t. \tag{2}$$

In addition, we assume the SMP exceeds the balancing-market-selling price and is less than the balancing-market-purchasing price ($\pi_t^{\downarrow} < \pi_t^s < \pi_t^{\uparrow}$), such that the retailer will avoid bidding in the real-time balancing

[34] See page 130, Table 5 of Cosmo et al. (2014). The authors estimated the impact of incentive stimuli on electricity consumption for different tariff groups. To estimate the average effects, we weight their estimates by number of households in each tariff group, divided by the average daily demand for off-peak, shoulder, and peak periods.

Table 12
Baseline *vs.* Demand-side Management (DSM) Stimuli.

	Profit (€)		Bill (€)		Demand	Market
	Profit	Δ^*	Bill	Δ^{**}	(kWh)	Gain (€)
Baseline	718.61	7.35	1199.34	0.00	8526	7.35
DSM Stimuli						
Monthly Bill	721.73	10.47	1199.34	0.00	8475	10.47
In-house Display	720.48	9.22	1199.34	0.00	8495	9.22
DSM Stimuli & Regulation						
Monthly Bill, $\delta = 0.995$	715.62	4.36	1193.34	6.00	8472	10.36
In-house Display, $\delta = 0.995$	714.37	3.11	1193.34	6.00	8492	9.11
Flat Tariff	711.26	–	1199.34	–	8506	–

Δ^* Difference between dynamic and flat tariff; Δ^{**} difference between flat and dynamic tariff.

market. This means that the expected amount of energy purchased from the balancing market is zero i.e., $\mathbb{E}[\Delta E_t^{\uparrow}] = \mathbb{E}[\Delta E_t^{\downarrow}] = 0$.

If the forecast for the next day's demand differs from the actual demand ($\mathbb{E}[d_t] \neq d_t$), either $\Delta E_t^{\uparrow}$ or $\Delta E_t^{\downarrow}$ will be positive, resulting in monetary losses of the form

$$Loss = (\pi_t^{\uparrow} - \pi_t^{s})\Delta E_t^{\uparrow} + (\pi_t^{s} - \pi_t^{\downarrow})\Delta E_t^{\downarrow}. \quad (20)$$

The first (second) component of (20) denotes the loss from buying (selling) on the balancing market. The greater the volume of balancing market trading ($\Delta E_t^{\uparrow}$ or $\Delta E_t^{\downarrow}$), the greater the monetary losses from forecast errors.

To estimate the loss in Eq. (20), we need to simulate $\Delta E_t^{\uparrow}$ and $\Delta E_t^{\downarrow}$, as well as $\pi_t^{\uparrow}$ and $\pi_t^{\downarrow}$. (Data on day-ahead SMPs in Ireland in 2010 gives π_t^{s}.) Note that the retailer buys electricity from the day-ahead market for an amount equalling to their forecast of the consumer's electricity demand, or $E_t^s = \mathbb{E}[d_t] = d_t + e_t$. The market balancing constraint requires $e_t = \Delta E_t^{\uparrow} - \Delta E_t^{\downarrow}$. If e_t is positive the retailer sells the extra electricity at the balancing market, resulting in $\Delta E_t^{\uparrow} = e_t$ and $\Delta E_t^{\downarrow} = 0$. If e_t is negative, the retailer will purchase more electricity from the balancing market, resulting in $\Delta E_t^{\uparrow} = 0$ and $\Delta E_t^{\downarrow} = e_t$. Therefore, to simulate $\Delta E_t^{\uparrow}$ and $\Delta E_t^{\downarrow}$, we first simulate forecasts of consumer demand for a representative household for 2010, while assuming a given mean absolute percentage error (MAPE)[35]. We generate a forecast error e_t randomly from a normal distribution:

$$e_t \sim N\left[0, \left(\eta \cdot d_t\right)^2\right],$$

where d_t denotes the actual electricity demand and $\eta = \text{MAPE} \cdot \sqrt{\pi/2}$.[36]

To estimate $\pi_t^{\uparrow}$ and $\pi_t^{\downarrow}$, we examine the relationship between the historical day-ahead and real-time prices, and find that the balancing-market-selling (purchasing) price is about 15% lower (higher) than the day-ahead price, or $\pi_t^{\uparrow} = 1.15\pi_t^{s}$ and $\pi_t^{\downarrow} = 0.85\pi_t^{s}$.[37]

Given π_t^{s} and the simulated $\Delta E_t^{\uparrow}$, $\Delta E_t^{\downarrow}$, $\pi_t^{\uparrow}$, and $\pi_t^{\downarrow}$, we can estimate the retailer's monetary loss due to forecast errors on the representative households' electricity demand. The loss is then aggregated into annual, which is reported in Table 13. We then report the corresponding losses for values of MAPE (of simulated demand forecast) equalling 10%, 5%, 2%, and 1%.

Table 13
Forecast errors and retailers' lost profit.

	MAPES of demand forecast			
	10%	5%	2%	1%
Lost Profit (€/year)	7.22	3.61	1.44	0.72

A 1% reduction in the MAPE (i.e. an improvement in the forecast accuracy) is accompanied by an average profit gain of €0.72/year from the representative household, or about €3.9 million/year for a major electricity supplier like British Gas. This therefore explains why every year electricity retailers invest substantially into improving forecast accuracy of the day-ahead load. In a world with high renewable penetration, the real-time balancing prices becomes more volatile, further boosting the monetary gain from improving forecast accuracy.

8. Conclusions and remarks

The increasing penetration of renewable energy challenges the stability and flexibility of the power system, with the use of dynamic tariffs a possible solution. Dynamic tariffs facilitate the alignment of retail and spot market prices, transferring a component of retailer-centred risks to consumers. By offering consumers dynamic electricity tariffs, there is the expectation that consumers will respond to pricing signals, reallocating demand from periods with low renewable and high system load, to periods with high renewable and low system load.

In this paper we evaluate the impact of dynamic tariffs on the retail market in terms of retail profit, household's electricity demand, household's electricity bill, as well as the market gain relative to a flat tariff. To do this we model the electricity retail market where an electricity retailer sets dynamic tariffs to maximize its expected profit, and a representative household responds to the dynamic tariffs in order to maximize its own utility. Before setting the tariff, a rational retailer needs to anticipate the response of households, such that the profit-maximization dynamic tariff is based upon household's responses.

Our results suggest that although dynamic tariffs bring market gains without proper market regulation all gains would go to the retailer. This is because to maximize profit, the retailer will ensure the household to be indifferent between dynamic and flat tariffs. That said, market regulation that cares about consumer welfare may result in a transfer of the market gain from the retailer to the household. This can be manifested as targeting on a more competitive and regulated market, such as easing the licencing process to become an electricity retailer. As a result, both market entities are better-off.

The results also show that the market gain is positively related to households' price elasticities of demand. This is because more elastic households would be more sensitive to pricing signals, which facilitates the retailer's arbitrage activity from buying low and selling high.

[35] Forecast error is usually measured by MAPE - the average of absolute percentage errors by which the forecasts differ from actual values.

[36] We need to find the value of η such that $\mathbb{E}[|e_t|] = \text{MAPE} \cdot d_t$. Given that the expected absolute value of a normal random variable with mean 0 and standard deviation η is $\eta \cdot \sqrt{2/\pi}$, we solve for η that satisfies $\text{MAPE} = \eta \cdot \sqrt{2/\pi}$.

[37] The real-time balancing prices for Ireland are not publicly available. However, given the similarity between the GB and Irish electricity market in terms of similar fuel mixes and operation rules, we use GB historical data to simulate $\pi_t^{\uparrow}$ and $\pi_t^{\downarrow}$ in Ireland. In GB in 2013 the average SMP was £50.15/MWh, the average balancing-market-purchasing price was £58.07/MWh, and the average balancing-market-selling price was £43.93/MWh. On average the balancing-market-purchasing price is 15.8% higher than the SMP, and the balancing-market-selling price is 12.4% lower than the SMP. Therefore, the assumption that the balancing price was 15% lower or higher than the SMP seems to be plausible.

Therefore, in a future scenario with the adoption of smart household appliances, consumers will be more price-sensitive, such that dynamic tariffs may be more attractive from policymakers' perspective as the efficiency gain would also increase.

We also find that the introduction of dynamic tariffs do not guarantee a reduction in households' electricity demand. What really matter is average electricity retail prices under dynamic tariffs relative to the flat tariff: if the former is lower than the latter, households may consume more electricity under dynamic tariffs. Put another way, although the dynamic tariff scheme results in a more efficient retail market from aligning retail and wholesale prices, it may not be an effective tool for energy conservation.

Our model could be further improved. For example, the assumption that households have identical load profiles and elasticities can be relaxed, such that the retailer may wish to impose different dynamic tariff schemes on different types of households, with households choosing between different dynamic tariff schemes. This will complicate our model as it imposes more restrictions on the retailer's objective function, but will certainly result in a more efficient market due to market segmentation.

CRediT authorship contribution statement

Bowei Guo: Conceptualization, Methodology, Software, Data curation, Writing – original draft, Visualization, Investigation, Validation, Writing – review & editing. **Melvyn Weeks:** Conceptualization, Methodology, Writing – original draft, Investigation, Supervision, Validation, Writing – review & editing.

Appendix. Solution method

Model (16) is a non-linear programming (NLP) problem, more specifically it is a quadratically constrained quadratic programming (QCQP) problem. Since both the objective function and the inequality constraints are convex and twice continuously differentiable, the barrier method (Boyd and Vandenberghe, 2004) is used for solving the problem.

The goal is to approximately formulate the inequality constrained problem as an equality constrained problem so that Newton's method can be applied. First, the model can be rearranged as:

$$\begin{aligned}
\min \quad & -\mathbb{E}[\phi(\mathbf{x})] \\
\text{subject to} \quad & \mathbb{E}[d_t] = E_t^s, \quad \forall t \\
& \mathbb{E}[d_t] = \mathbb{E}[d_{0,t}]\left(1 + \epsilon_{t,t}\frac{\pi_t - \pi_0 + r_t}{\pi_0} + \sum_{t'=1}^{T}\epsilon_{t,t'}\frac{\pi_{t'} - \pi_0 + r'_t}{\pi_0}\right)\cdot \kappa_t, \quad \forall t \\
& D^{\text{Min}} - \sum_{t=1}^{T}\mathbb{E}[d_t] \le 0, \\
& \sum_{t=1}^{T}\mathbb{E}[d_t] - D^{\text{Max}} \le 0, \\
& \pi_t^{\text{Min}} - \pi_t \le 0, \quad \forall t \\
& \pi_t - \pi_t^{\text{Max}} \le 0, \quad \forall t \\
& \sum_{t=1}^{T}\mathbb{E}[d_t]\pi_t - \delta\cdot\sum_{t=1}^{T}\mathbb{E}[d_{0,t}]\pi_0 \le 0.
\end{aligned}$$

Then we can make the inequality constraints implicit in the objective function:

$$\begin{aligned}
\min \quad & -\mathbb{E}[\phi(\mathbf{x})] + I_(D^{\text{Min}} - \sum_{t=1}^{T}\mathbb{E}[d_t]) + I_(\sum_{t=1}^{T}\mathbb{E}[d_t] - D^{\text{Max}}) \\
& + \sum_{t=1}^{T} I_(\pi_t^{\text{Min}} - \pi_t) + \sum_{t=1}^{T} I_(\pi_t - \pi_t^{\text{Max}}) \\
& + I_(\sum_{t=1}^{T}\mathbb{E}[d_t]\pi_t - \delta\cdot\sum_{t=1}^{T}\mathbb{E}[d_{0,t}]\pi_0)
\end{aligned} \tag{21}$$

Table A.1
Algorithm: barrier method.

Given
starting with a value $\lambda := \lambda^{(0)} > 0$, solve the barrier problem (21) using Newton's method to get $\mathbf{x}^{(0)} := \mathbf{x}^{\star}(\lambda)$.
Do
for barrier parameter $\mu > 1$, update $\lambda^{(1)} = \mu\lambda^{(0)}$ for $i = 1, 2, \ldots,$ do:
1. solve the barrier problem at $\lambda := \lambda^{(i)}$ using Newton's method initialized at $\pi_t^{(i-1)}$ to produce $\mathbf{x}^{(i)} := \mathbf{x}^{\star}(\lambda^{(i)})$;
2. stop if $\frac{m}{\lambda} \le \varepsilon$, where m is the number of inequalities and ε is a desired level of accuracy;
3. Else, update $\lambda^{(i+1)} = \mu\lambda^{(i)}$.

$$\begin{aligned}
\text{subject to} \quad & \mathbb{E}[d_t] = E_t^s, \quad \forall t \\
& \mathbb{E}[d_t] = \mathbb{E}[d_{0,t}]\left(1 + \epsilon_{t,t}\frac{\pi_t - \pi_0 + r_t}{\pi_0} + \sum_{t'=1}^{T}\epsilon_{t,t'}\frac{\pi_{t'} - \pi_0 + r'_t}{\pi_0}\right)\cdot \kappa_t, \quad \forall t
\end{aligned}$$

where $I\_(\cdot) : \mathbf{R} \to \mathbf{R}$ is the indicator function for the non-positive reals,

$$I\_(u) = \begin{cases} 0 & u \le 0 \\ \infty & u > 0. \end{cases}$$

Then, when $u \le 0$, i.e. the inequality constraint holds, $I\_(u)$ equals 0, adding no penalty to (21). However, when $u > 0$, i.e. the inequality constraint fails to hold, $I\_(u)$ equals infinity, boosting (21) to infinity as well. Hence while minimize (21), the inequality constraint would always hold.

Note that since here the indicator function is not differentiable, we approximate the indicator function $I\_(\cdot)$ by $\hat{I}\_(\cdot)$:

$$\hat{I}\_(u) = -(1/\lambda)\log(-u).$$

The accuracy of the approximation depends on the value of λ, a positive index number. As λ increases, the approximation becomes more accurate. Similar to $I\_(\cdot)$, $\hat{I}\_(\cdot)$ is convex, non-decreasing, and takes the value ∞ when u is greater than 0. However, unlike $I\_(\cdot)$, $\hat{I}\_(\cdot)$ is differentiable and closed: it increases to ∞ as u increases from negative to 0.

Then, we substitute $I\_(\cdot)$ by $\hat{I}\_(\cdot)$ and obtain

$$\begin{aligned}
\min \quad & -\mathbb{E}[\phi(\mathbf{x})] + \frac{1}{\lambda}g(\mathbf{x}) \\
\text{subject to} \quad & \mathbb{E}[d_t] = E_t^s, \quad \forall t \\
& \mathbb{E}[d_t] = \mathbb{E}[d_{0,t}]\left(1 + \epsilon_{t,t}\frac{\pi_t - \pi_0 + r_t}{\pi_0} + \sum_{t'=1}^{T}\epsilon_{t,t'}\frac{\pi_{t'} - \pi_0 + r'_t}{\pi_0}\right)\cdot \kappa_t, \quad \forall t,
\end{aligned}$$

where $g(\mathbf{x})$ is the logarithm barrier, and

$$\begin{aligned}
g(\mathbf{x}) = & -\log(-D^{\text{Min}} + \sum_{t=1}^{T}\mathbb{E}[d_t]) - \log(-\sum_{t=1}^{T}\mathbb{E}[d_t] + D^{\text{Max}}) \\
& - \sum_{t=1}^{T}\log(-\pi_t^{Min} + \pi_t) \\
& - \sum_{t=1}^{T}\log(-\pi_t + \pi_t^{\text{Max}}) - \log(-\sum_{t=1}^{T}\log[d_t]\pi_t + \delta\cdot\sum_{t=1}^{T}\mathbb{E}[d_{0,t}]\pi_0).
\end{aligned}$$

Then Newton's method can be implemented to solve this. If we slightly simplify the notation and multiply the objective function by λ, it becomes

$$\min \quad -\lambda\mathbb{E}[\phi(\mathbf{x})] + g(\mathbf{x})$$

The quality of approximation improves as λ grows. When λ is large, however, the objective function $-\lambda\phi(\mathbf{x}) + g(\mathbf{x})$ is hard to minimize by Newton's method because its Hessian varies rapidly near the boundary. As a result, the barrier method solves a sequence of the problem by increasing λ at each step, and starts the Newton minimization at the solution of the problem for the previous value of λ. Formally, the barrier method can be summarized as Table A.1 (Boyd and Vandenberghe, 2004).

From Algorithm 1, the barrier method is a double iterative algorithm — we have the outer iteration when we gradually increase λ by a factor μ, and compute $\mathbf{x}^{\star}(\lambda)$ from the previously computed $\mathbf{x}^{\star}(\lambda)$; and the inner iteration when we apply the Newton process to compute $\mathbf{x}^{\star}(\lambda)$.

In Section 6, we use AIMMS, a high-level modelling software to solve the NLP problem.

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